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Research Article

A Comparative Analysis of Some Almost Unbiased Modified Ratio Estimators for Population Mean with Known Population Parameters of the Auxiliary Variable.

Etaga Harrison O[1]. Nwosu Chinwe R[2] and Udechukwu Chiamaka U[3].

[1,2,3] *Department of Statistics, Nnamdi Azikiwe University, P.M.B 5025 Awka. Anambra State, Nigeria*
Corresponding Author: (etagaomavua@yahoo.com)

ABSTRACT: *This Paper deals with a comparative analysis of some almost unbiased modified ratio estimators for population mean with known population parameters. Over the years a lot of modified estimators have been proposed using information from auxiliary variable that is highly related to the studied variable. We compared some of the estimators and studied their behaviors at varying sample sizes. It was observed that some of the estimators are distribution sensitive.*

Keywords: *Ratio estimator, Auxiliary information, Mean Square Error, Efficiency, Bias, Population Mean.*

I. INTRODUCTION

In sample surveys, auxiliary information on the finite population under study is quite often available from previous experience, census or administrative databases. Sampling theory describes a wide variety of techniques/methods for using auxiliary information to improve the sampling design and to obtain more efficient estimators like Ratio, Product, and Regression estimators. It is believed that Ratio estimators improves the precision of estimate of the population mean or total of a study variable by using prior information on auxiliary variable X which is corrected with the study variable Y [1],[2]. Many authors have proposed one estimator or the other for the population mean. [3] proposed and improved estimators of the population mean using the correlation coefficient. His estimators are :

$$\overline{Y}_{pr1} = \frac{\bar{y}}{(\bar{x}\, C_x + \rho)}(\bar{X}\, C_x + \rho) \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

$$\overline{Y}_{pr2} = \frac{\bar{y}}{(\bar{x}\, \rho + C_x)}(\overline{X\rho} + C_x) \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

$$\overline{Y}_{pr3} = \frac{\bar{y}}{(\bar{x}\, \beta_2(x) + \rho)}(\bar{X}\, \beta_2(x) + \rho) \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$$\overline{Y}_{pr4} = \frac{\bar{y}}{(\bar{x}\, \rho + \beta_2(x))}(\bar{X}\, \rho + \beta_2(x)) \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

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It was observed by [4] that when the study variable y is highly correlated with the auxiliary variable, the use of auxiliary information in the ratio and product estimators can increase the precision of the estimates. Two classes of estimator proposed by [5] for the population mean of the study character using multi-auxiliary characters with known population means in presence of non-response. They also observed that the problem of estimating the population mean using multi-auxiliary character with known population means has been considered by several authors when there is no non-response on sample units. [6] while working on Ratio-cum-product estimator of finite population mean using known coefficient of variation and coefficient of kurtosis observed that in sample surveys, auxiliary information is used at both selection as well as estimation stages to improve the efficiency of the estimators. The authors are of the view that when the correlation between study variate and auxiliary variate is positive (high), the ratio method of estimation is used for estimating the population mean. They are also of the view that if the correlation is negative, the product method of estimation envisaged is used. [7] used coefficient of variation along with the population mean of auxiliary variate. Use of coefficient of kurtosis of auxiliary variate has also been in practice for improving the efficiency of the estimators of finite population mean. [8] and [9] utilized coefficient of kurtosis of auxiliary variate for estimating the finite population mean.
In all these works, all efforts are made to develop more efficient estimators by modifying the already existing ones. More recently, some authors have come up with some challenging estimators that are either a linear combination of others or product of some estimators. [10] worked on modified ratio estimators of population mean using linear combination of co-efficient of skewness and quartile deviation. These various estimators have been classified in class one, two and class three based on the type of constant used in the estimation. Jeelani et al observed that Sampling is not mere substitution of a partial coverage for a total coverage. They also observed that most of the times in sample surveys, along with the variable of interest Y, information on auxiliary X, which is highly correlated with Y is also collected. That this information on auxiliary variable may well be utilized to obtain a more efficient estimator of population mean. Ratio method of estimation is therefore one of such example which utilizes the information of auxiliary variable X which is positively correlated with the variable of interest Y, in order to improve the precision of the estimate of population mean. Improvement in the precision of the estimator through the use of Ratio method of estimation are achieved by introducing a large number of modified Ratio estimators which utilizes the information on known values of Co-efficient of variation, Co-efficient of kurtosis, Co-efficient of skewness etc. [11] gave some situations where information on auxiliary variables may be applied. Such examples as, tonnage (or seat capacity) of each vehicle or ship is known in survey sampling of transportation, number of beds in different hospitals may be known in hospital surveys, number of polluting industries and vehicles is known in environmental surveys, nature of employment status, educational status, food availability and medical aids of a locality is well known in advance for estimating various demographic parameters in demographic surveys. [12] proposed a third class of estimators in which they used the deciles of the auxiliary variable as information to estimate the population mean. In their work, they computed ten deciles and used them to estimate the population mean, computed their bias as well as the mean square error. They observed that as the deciles values increases (D_1 to D_{10}), the Mean Squares decreases showing that the higher deciles are more efficient than the lower deciles.

1.1 Aims and Objectives.

Thus motivated by the various discussions/conclusions of the various authors, we decided to carry out a comparative analysis of some of the estimators reviewed, and evaluate their performance based on Biased and Mean Square error of the estimators through simulated data with a view to ascertaining the correctness or otherwise of the authors conclusions.

II. METHODOLOGY

The assigned codes to authors whose works were reviewed are presented in Table 1, while the estimators, Bias Mean Square Error and Constants of the three classes of estimators are presented in the Tables 2, 3, and 4.

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Table1. Authors assigned codes.

Assigned code	**Estimators (Authors)**	**Year**
A	Sisodia and Dwivedi	1981
B	Singh et.al	2004,2005
C	Yan and Tian	2010
D	Singh and Tailor	2003
E	Upadhyaya and Singh	1999
F	Upadhyaya and Singh	1999
G	Yan and Tian	2010
H	Yan and Tian	2010
I	Subramani and Kumarapandiyan	2012
J	Subramani and Kumarapandiyan	2012
K	Etaga and Udechukwu	2013
L	Kadilar and Cingi	2004
M	Kadilar and Cingi	2004
N	Kadilar and Cingi	2004
O	Kadilar and Cingi	2004
P	Kadilar and Cingi	2004
Q	Yan and Tian	2010
R	Kadilar and Cingi	2006
S	Kadilar and Cingi	2006
T	Kadilar and Cingi	2006
U	Kadilar and Cingi	2006
V	Kadilar and Cingi	2006
W	Subramani and Kumarapandiyan	2012
X	Etaga and Udechukwu	2013
Y	Subramani and Kumarapandiyan	2013
Z	Subramani and Kumarapandiyan	2013
AA	Subramani and Kumarapandiyan	2013
AB	Subramani and Kumarapandiyan	2013
AC	Subramani and Kumarapandiyan	2013
AD	Subramani and Kumarapandiyan	2013
AE	Subramani and Kumarapandiyan	2013
AF	Subramani and Kumarapandiyan	2013
AG	Subramani and Kumarapandiyan	2013

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Table 2. Existing modified ratio type estimators (class 1) by Author's Code with their biases, mean squared errors and their constants.

Author's codes	Estimator	Bias	MSE	Constants
A	$\hat{\bar{Y}}_1 = \bar{y}\,[\frac{\bar{X}+C_x}{\bar{x}+C_x}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_1^2 C_x^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\bar{X}}{\bar{X}+C_x}$
B	$\hat{\bar{Y}}_2 = \bar{y}\,[\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\bar{X}}{\bar{X}+\beta_2}$
C	$\hat{\bar{Y}}_3 = \bar{y}\,[\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\bar{X}}{\bar{X}+\beta_1}$
D	$\hat{\bar{Y}}_4 = \bar{y}\,[\frac{\bar{X}+\rho}{\bar{x}+\rho}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_4^2 C_x^2 - \theta_4 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_x C_y \rho)$	$\theta_4 = \frac{\bar{X}}{\bar{X}+\rho}$
E	$\hat{\bar{Y}}_5 = \bar{y}\,[\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_5^2 C_x^2 - \theta_5 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_x C_y \rho)$	$\theta_5 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}$
F	$\hat{\bar{Y}}_6 = \bar{y}\,[\frac{\bar{X}\beta_2+C_x}{\bar{x}\beta_2+C_x}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_6^2 C_x^2 - \theta_6 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_x C_y \rho)$	$\theta_6 = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+C_x}$
G	$\hat{\bar{Y}}_7 = \bar{y}\,[\frac{\bar{X}\beta_1+\beta_2}{\bar{x}\beta_1+\beta_2}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_7^2 C_x^2 - \theta_7 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_x C_y \rho)$	$\theta_7 = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+\beta_2}$
H	$\hat{\bar{Y}}_8 = \bar{y}\,[\frac{\bar{X}C_x+\beta_1}{\bar{x}C_x+\beta_1}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_8^2 C_x^2 - \theta_8 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_8^2 C_x^2 - 2\theta_8 C_x C_y \rho)$	$\theta_8 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_1}$
I	$\hat{\bar{Y}}_9 = \bar{y}\,[\frac{\bar{X}+M_d}{\bar{x}+M_d}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_9^2 C_x^2 - \theta_9 C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_9^2 C_x^2 - 2\theta_9 C_x C_y \rho)$	$\theta_9 = \frac{\bar{X}}{\bar{X}+M_d}$
J	$\hat{\bar{Y}}_{10} = \bar{y}\,[\frac{\bar{X}\beta_1+M_d}{\bar{x}\beta_1+M_d}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_{10}^2 C_x^2 - \theta_{10} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_{10}^2 C_x^2 - 2\theta_{10} C_x C_y \rho)$	$\theta_{10} = \frac{\bar{X}\beta_1}{\bar{X}\beta_1+M_d}$
K	$\hat{\bar{Y}}_{11} = \bar{y}\,[\frac{\bar{X}\beta_2+\beta_1}{\bar{x}\beta_2+\beta_1}]$	$\frac{(1-f)}{n}\bar{Y}\,(\theta_{11}^2 C_x^2 - \theta_{11} C_x C_y \rho)$	$\frac{(1-f)}{n}\bar{Y}^2\,(C_y^2 + \theta_{11}^2 C_x^2 - 2\theta_{11} C_x C_y \rho)$	$\theta_{11} = \frac{\bar{X}\beta_2}{\bar{X}\beta_2+\beta_1}$

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Table 3. Existing modified ratio type estimators (class 2) by their authors with their biases, mean squared errors and their constants.

Author's codes	Estimator	Bias	MSE	Constants
L	$\bar{Y}_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X}$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{12}^2$	$\frac{(1-f)}{n}(R_{12}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{12}=\frac{\bar{Y}}{\bar{X}}$
M	$\bar{Y}_{13}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{13}^2$	$\frac{(1-f)}{n}(R_{13}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{13}=\frac{\bar{Y}}{\bar{X}+C_x}$
N	$\bar{Y}_{14}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{14}^2$	$\frac{(1-f)}{n}(R_{14}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{14}=\frac{\bar{Y}}{\bar{X}+\beta_2}$
O	$\bar{Y}_{15}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{15}^2$	$\frac{(1-f)}{n}(R_{15}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{15}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+C_x}$
P	$\bar{Y}_{16}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{16}^2$	$\frac{(1-f)}{n}(R_{16}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{16}=\frac{\bar{Y}C_x}{\bar{X}C_x+\beta_2}$
Q	$\bar{Y}_{17}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{17}^2$	$\frac{(1-f)}{n}(R_{17}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{17}=\frac{\bar{Y}}{\bar{X}+\beta_1}$
R	$\bar{Y}_{18}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\rho)}(\bar{X}+\rho)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{18}^2$	$\frac{(1-f)}{n}(R_{18}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{18}=\frac{\bar{Y}}{\bar{X}+\rho}$
S	$\bar{Y}_{19}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{19}^2$	$\frac{(1-f)}{n}(R_{19}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{19}=\frac{\bar{Y}C_x}{\bar{X}C_x+\rho}$
T	$\bar{Y}_{20}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{20}^2$	$\frac{(1-f)}{n}(R_{20}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{20}=\frac{\bar{Y}\rho}{\bar{X}\rho+C_x}$
U	$\bar{Y}_{21}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{21}^2$	$\frac{(1-f)}{n}(R_{21}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{21}=\frac{\bar{Y}\beta_2}{\bar{X}\beta_2+\rho}$
V	$\bar{Y}_{22}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{22}^2$	$\frac{(1-f)}{n}(R_{22}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{22}=\frac{\bar{Y}\rho}{\bar{X}\rho+\beta_2}$
W	$\bar{Y}_{23}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+M_d)}(\bar{X}\beta_1+M_d)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{23}^2$	$\frac{(1-f)}{n}(R_{23}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{23}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_1+M_d}$
X	$\bar{Y}_{24}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\beta_1)}(\bar{X}\beta_2+\beta_1)$	$\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{24}^2$	$\frac{(1-f)}{n}(R_{24}^2 s_x^2+s_y^2(1-\rho^2))$	$R_{24}=\frac{\bar{Y}\beta_1}{\bar{X}\beta_2+\beta_1}$

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Where N = Population Size; n = sample size; f = n/N (sampling fraction); Y = study variable; X = Auxiliary variable; $\bar{X}$, $\bar{Y}$ = Population means; $\bar{x}$, $\bar{y}$ = Sample means; S_x,S_y = Population standard deviations; C_x, C_y = Coefficient of variation; ρ = co-efficient of correlation; β_1 = Coefficient of skewness of the auxiliary variable; β_2 = Coefficient of kurtosis of the auxiliary variable; M_d = Median of the auxiliary variable; B (.) = bias of the estimator; MSE(.) = Mean Squared Error of the estimator; θ and R = constants; d(.) = deciles and $Y_{(.)}$ = estimator.

Table 4. Existing modified ratio type estimators (class 3) by their authors with their biases, mean squared errors and their constants.

Author's codes	Estimator	Bias	MSE	Constants
Y	$\bar{Y}_{p1}= \bar{y}\ [\frac{\bar{X}+D_1}{\bar{x}+D_1}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p1}{}^2C_x{}^2 - \theta_{p1}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p1}{}^2C_x{}^2-2\theta_{p1}C_xC_y\rho)$	$\theta_{p1}=\frac{\bar{X}}{\bar{X}+D_1}$
Z	$\bar{Y}_{p2}= \bar{y}\ [\frac{\bar{X}+D_2}{\bar{x}+D_2}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p2}{}^2C_x{}^2 - \theta_{p2}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p2}{}^2C_x{}^2-2\theta_{p2}C_xC_y\rho)$	$\theta_{p2}=\frac{\bar{X}}{\bar{X}+D_2}$
AA	$\bar{Y}_{p3}= \bar{y}\ [\frac{\bar{X}+D_3}{\bar{x}+D_3}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p3}{}^2C_x{}^2 - \theta_{p3}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p3}{}^2C_x{}^2-2\theta_{p3}C_xC_y\rho)$	$\theta_{p3}=\frac{\bar{X}}{\bar{X}+D_3}$
AB	$\bar{Y}_{p4}= \bar{y}\ [\frac{\bar{X}+D_4}{\bar{x}+D_4}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p4}{}^2C_x{}^2 - \theta_{p4}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p4}{}^2C_x{}^2-2\theta_{p4}C_xC_y\rho)$	$\theta_{p4}=\frac{\bar{X}}{\bar{X}+D_4}$
AC	$\bar{Y}_{p5}= \bar{y}\ [\frac{\bar{X}+D_5}{\bar{x}+D_5}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p5}{}^2C_x{}^2 - \theta_{p5}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p5}{}^2C_x{}^2-2\theta_{p5}C_xC_y\rho)$	$\theta_{p5}=\frac{\bar{X}}{\bar{X}+D_5}$
AD	$\bar{Y}_{p6}= \bar{y}\ [\frac{\bar{X}+D_6}{\bar{x}+D_6}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p6}{}^2C_x{}^2 - \theta_{p6}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p6}{}^2C_x{}^2-2\theta_{p6}C_xC_y\rho)$	$\theta_{p6}=\frac{\bar{X}}{\bar{X}+D_6}$
AE	$\bar{Y}_{p7}= \bar{y}\ [\frac{\bar{X}+D_7}{\bar{x}+D_7}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p7}{}^2C_x{}^2 - \theta_{p7}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p7}{}^2C_x{}^2-2\theta_{p7}C_xC_y\rho)$	$\theta_{p7}=\frac{\bar{X}}{\bar{X}+D_7}$
AF	$\bar{Y}_{p8}= \bar{y}\ [\frac{\bar{X}+D_8}{\bar{x}+D_8}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p8}{}^2C_x{}^2 - \theta_{p8}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p8}{}^2C_x{}^2-2\theta_{p8}C_xC_y\rho)$	$\theta_{p8}=\frac{\bar{X}}{\bar{X}+D_8}$
AG	$\bar{Y}_{p9}= \bar{y}\ [\frac{\bar{X}+D_9}{\bar{x}+D_9}]$	$\frac{(1-f)}{n}\ \bar{Y}\ (\theta_{p9}{}^2C_x{}^2 - \theta_{p9}C_xC_y\rho)$	$\frac{(1-f)}{n}\ \bar{Y}^2\ (C_y{}^2+\theta_{p9}{}^2C_x{}^2-2\theta_{p9}C_xC_y\rho)$	$\theta_{p9}=\frac{\bar{X}}{\bar{X}+D_9}$

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III. EMPIRICAL STUDY

In order to comparatively discuss the efficiency of the selected modified estimators, five populations were simulated using Minitab. We simulated from the Normal, Chi-square, Exponential, Poisson and Binomial distributions. Then a hypothesized population of size 230 was simulated from each of the distributions (population) out of which 50 was sampled using Simple Random Sampling (SRS). The various parameters and constants in the three classes were then estimated and are tabulated in Table 5 and 6.

Table 5. Parameters for the five populations for the various estimators

	POP1	POP2	POP3	POP4	POP5
Parameters/ Constants	Normal	Chi-square	Exponential	Poisson	Binomial
N	230	230	230	230	230
N	50	50	50	50	50
Y	4.063	10.047	3.224	6.943	2.5348
X	4.0262	9.983	3.045	6.778	2.5435
P	0.992	0.993	0.953	0.978	0.941
S_y	1.0123	4.045	2.998	2.844	1.1316
C_Y	24.92	40.26	93	40.96	44.64
S_X	0.9939	4.456	3.427	2.742	1.1196
C_X	24.69	44.64	112.57	40.45	44.02
B_2	0.09	0.57	23.95	0.24	-0.09
B_1	0.32	0.78	3.68	0.57	-0.31
M_d	4.0059	9.428	2.1	6	3
f=n/N	0.217391	0.217391	0.217391	0.217391	0.217391
d1	2.236	4.2	0.527975	3.258824	0.885714
d2	2.646939	5.985714	1.035951	4.32	1.897297
d3	2.928571	7.440984	1.543926	5.24	2.083784
d4	3.203846	8.421311	2.051902	6.021538	2.27027
d5	9.293548	26.87465	2.559877	6.587692	3.054545
d6	3.74186	10.47037	3.067853	7.153846	3.233766
d7	4.062791	11.57778	3.575828	8.17037	3.412987
d8	4.383721	13.20435	5.132	9.45	3.592208
d9	4.797143	16.2	6.788	10.6	3.93

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Table 6. Constants of the five populations for the various estimators by authors

constant ϴ

	POP1	POP2	POP3	POP4	POP5
Author's codes	**Normal**	**Chi-square**	**Exponential**	**Poisson**	**Binomial**
A	0.140207	0.182762	0.026337	0.143517	0.054624
B	0.978135	0.945987	0.112799	0.965802	1.036682
C	0.926372	0.927529	0.452788	0.922428	1.138796
D	0.80232	0.90953	0.761631	0.873904	0.729947
E	0.999095	0.998723	0.934692	0.999125	1.000804
F	0.014464	0.113059	0.393146	0.038661	-0.00523
G	0.934706	0.931792	0.318743	0.941513	0.897551
H	0.996791	0.998253	0.989378	0.997925	1.002776
I	0.501264	0.514296	0.591837	0.530443	0.458826
J	0.243354	0.45233	0.842172	0.391694	-0.35654
K	0.531038	0.879449	0.951963	0.740522	0.424770
Y	0.642937	0.703871	0.852231	0.675313	0.741715
Z	0.603344	0.62516	0.74615	0.610741	0.572758
AA	0.578912	0.572946	0.663554	0.563987	0.549675
AB	0.556871	0.542427	0.597422	0.52955	0.52838
AC	0.302273	0.270853	0.543277	0.507119	0.454355
AD	0.518302	0.488086	0.498131	0.486511	0.44026
AE	0.497738	0.463017	0.459912	0.453427	0.427013
AF	0.478744	0.430537	0.372386	0.417673	0.414541
AG	0.456312	0.381278	0.309672	0.390033	0.39291
		Constants R			
L	1.00914	1.006411	1.058785	1.024343	0.99658
M	0.141488	0.183934	0.027886	0.14701	0.054437
N	0.987075	0.952052	0.11943	0.989313	1.033136
O	0.014596	0.113784	0.416257	0.039602	-0.00521
P	1.008227	1.005125	0.989638	1.023448	0.997381
Q	0.93484	0.933476	0.479405	0.944883	1.1349

R	0.809653	0.915361	0.806403	0.895178	0.72745
S	0.999169	1.004173	1.055849	1.020702	0.988274
T	0.140514	0.18288	0.026608	0.144231	0.051391
U	0.269995	0.856879	1.045127	0.639731	-0.32037
V	0.986901	0.951689	0.114423	0.988553	1.035518
W	0.245578	0.45523	0.891679	0.401229	-0.35532
X	1.905393	1.21117	0.154871	1.801554	1.458093

The bias and Mean square errors of the estimator listed in Table 2, 3 and 4 for the five populations are then estimated and are as given in Tables 7 and 8.

Table 7. Bias of the Five Populations for the Various Estimators

	Bias (class 1)				
Estimators	Normal	Chi-square	Exponential	Poisson	Binomial
A	-4.68007	-40.8241	-12.8164	-21.6097	-3.77805
B	-0.87616	14.94629	-48.6538	-4.2123	6.569661
C	-2.68874	9.289811	-96.8617	-11.1373	16.15696
D	-6.18718	3.97927	-12.5132	-18.0915	-12.5879
E	-0.0831	32.28424	88.08247	1.562413	3.581758
F	-0.55331	-27.724	-99.097	-6.54208	0.385606
G	-2.41095	10.57701	-95.508	-8.17269	-3.91274
H	-0.17195	32.12208	127.834	1.347591	3.740841
I	-9.71584	-61.4481	-73.9834	-43.376	-17.4762
J	-7.15	-62.8279	29.53783	-41.6936	35.92982
K	-9.67998	-4.44236	100.2235	-32.8931	-17.2912
	Bias (class 2)				
Estimators	Normal	Chi-square	Exponential	Poisson	Binomial
L	0.003875	0.031331	0.063918	0.017785	0.007687
M	7.62E-05	0.001047	4.43E-05	0.000366	2.29E-05
N	0.003708	0.028038	0.000813	0.016589	0.008262
O	8.11E-07	0.0004	0.009879	2.66E-05	2.1E-07
P	0.003868	0.031251	0.055842	0.017754	0.0077
Q	0.003326	0.026955	0.013104	31.04162	0.009969

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R	0.002495	0.025919	0.037078	0.013583	0.004096
S	0.003799	0.031192	0.063564	0.017659	0.00756
T	7.51E-05	0.001035	4.04E-05	0.000353	2.04E-05
U	0.000277	0.022713	0.06228	0.006937	0.000794
V	0.003706	0.028017	0.000747	0.016564	0.0083
W	0.00023	0.00641	0.045334	0.002729	0.000977
X	0.013816	0.045377	0.001368	0.055012	0.016456
	Bias (class 3)				
Y	-8.93067	-42.2834	35.37259	-37.8268	-12.1197
Z	-9.30678	-52.975	-19.6455	-41.2222	-16.7988
AA	-9.47823	-57.9253	-52.5176	-42.7551	-17.0973
AB	-9.59319	-60.0274	-72.5479	-43.3871	-17.3
AC	-8.19069	-53.0244	-84.7831	-43.5719	-17.4621
AD	-9.70374	-62.3254	-92.1183	-43.584	-17.3974
AE	-9.71553	-62.7617	-96.2906	-43.2875	-17.3089
AF	-9.6973	-62.7411	-98.8078	-42.5295	-17.2008
AG	-9.63975	-61.4483	-94.5862	-41.632	-16.9567

Table 8. The mean square error of the five populations for the various estimators

	MSE (Class 1)				
Estimators	Normal	Chi-square	Exponential	Poisson	Binomial
A	119.3325	1635.43	1323.048	940.369	180.6712
B	2.641168	43.73069	1067.167	55.82833	24.27438
C	3.439971	38.94345	359.8855	60.77781	29.58699
D	8.789739	36.341	130.5217	71.82001	32.75525
E	2.557801	69.22912	173.9344	55.18106	23.37258
F	155.9299	1963.584	449.4874	1173.181	202.3554
G	3.254359	39.85841	581.8274	58.02773	23.57685
H	2.560195	68.92465	213.329	55.15678	23.40911
I	41.93122	493.4125	207.946	316.1874	70.78277
J	93.03021	654.2723	135.3629	497.5033	357.7832
K	37.38129	36.54537	185.0437	132.1263	77.5848
Estimators	**MSE (Class 2)**				

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L	0.016001	0.318359	0.218985	0.12899	0.021781
M	0.000565	0.014087	0.013056	0.008052	0.002353
N	0.01532	0.285272	0.015535	0.120689	0.023237
O	0.000259	0.007597	0.044765	0.005694	0.002296
P	0.015973	0.317555	0.192948	0.128774	0.021813
Q	0.013768	0.274387	0.055162	0.110576	0.027566
R	0.010391	0.263978	0.132452	0.099813	0.012678
S	0.015692	0.316961	0.217844	0.128114	0.021458
T	0.000561	0.013967	0.013043	0.007957	0.002347
U	0.001383	0.231767	0.213703	0.053671	0.004309
V	0.015315	0.285058	0.01532	0.120512	0.023334
W	0.001188	0.067979	0.15907	0.024454	0.004772
X	0.05639	0.45948	0.01732	0.38746	0.04401
	MSE (Class 3)				
Y	22.77861	151.4264	137.8464	177.5967	31.75342
Z	27.49449	265.9442	132.6559	232.9692	51.31264
AA	30.65102	363.4343	160.7427	279.4869	54.84869
AB	33.65998	428.366	203.5086	317.202	58.29493
AC	79.51002	1264.47	251.9488	343.3432	71.64996
AD	39.29338	558.5019	301.5797	368.4547	74.43489
AE	42.48846	624.8047	350.1634	410.9615	77.12282
AF	45.55803	716.5935	484.1177	459.9376	79.71624
AG	49.32951	868.4746	599.524	499.9618	84.35765

IV. CONCLUSION

Concluding, we can see that the estimators may be distribution sensitive as some perform better in some distribution than others. This can be seen more clearly in Tables 9 to 11.

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Table 9: Ranks of class 1 estimators

SN	Estimators (MSE class 1)	Rank1	Rank2	Rank3	Rank4	Rank5
1	A	10	10	11	10	9
2	B	3	5	10	3	4
3	C	5	3	7	5	5
4	D	6	1	1	6	6
5	E	1	7	3	2	1
6	F	11	11	8	11	10
7	G	4	4	9	4	3
8	H	2	6	6	1	2
9	I	8	8	5	8	7
10	J	9	9	2	9	11
11	K	7	2	4	7	8

In the class 1 estimators, the estimator proposed by [E] ranked top in the Normal population followed by that by [H], then in the third place by [B]. In the second population, the Chi-square population, the estimator by [D] followed by our estimator [K] and then [C]. For the exponential population, [D] rank top followed by [J] and then [E]. In the Poisson population [H] top on the rank table followed by that by [E] and then that by [H] and lastly for the Binomial population [E] ranked top, followed by [H] and then [G].

Table 10: Ranks of class 2 estimators

SN	Estimators (MSE class 2)	Rank1	Rank2	Rank3	Rank4	Rank5
1	L	12	12	13	12	8
2	M	3	3	2	3	3
3	N	9	9	4	9	10
4	O	1	1	6	1	1
5	P	11	11	10	11	9
6	Q	7	7	7	7	12
7	R	6	6	8	6	6
8	S	10	10	12	10	7
9	T	2	2	1	2	2
10	U	5	5	11	5	4
11	V	8	8	3	8	11
12	W	4	4	9	4	5
13	X	13	13	5	13	13

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In the class 2 estimators most of which were proposed by [3],[13], the fourth estimator they proposed in 2004 rank top in Populations 1, 2, 4, and 5 . On the second place was their third estimator in 2006 while on the third place is their second estimator in 2004.

Table 11: Ranks of Class 3 Estimators

SN	Estimators (MSE class 3)	Rank1	Rank2	Rank3	Rank4	Rank5
d1	Y	1	1	2	1	1
d2	Z	2	2	1	2	2
d3	AA	3	3	3	3	3
d4	AB	4	4	4	4	4
d5	AC	9	9	5	5	5
d6	AD	5	5	6	6	6
d7	AE	6	6	7	7	7
d8	AF	7	7	8	8	8
d9	AG	8	8	9	9	9

For the third class of estimators which were entirely [12], it was observed that the lower deciles perform better than the higher ones in all populations. These results agree with the works of [12].

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